**\_\_\_\_\_\_\_Notes: Solving Quadratic Equations \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

A **quadratic function** is a function that can be written in the form y = ax2 + bx + c, where a0 Why not?

If the product of two numbers is *zero*, what do you know about the numbers? Why?

**Zero Product Property**: If $a∙b=0$, then either $a=0$ or $b=0$, or both.

This property can be used to solve **quadratic** **equations**. Quadratic means the variable is squared (*x*2). Quadratic equations can have up to 2 **solutions**. Sometimes the solutions are called **roots** or **zeros**.

**Steps for solving quadratic equations by factoring:**

1. Move all terms to one side of the equation so the equation is set to zero.
2. Factor the polynomial.
3. Set each factor equal to zero.
4. Solve each new equation.

**Examples:** Solve by factoring.

**1)** $x^{2}-3x-28=0$ **2)** $x^{2}+4x=12$ **3)** $x^{2}-9=0$

**4)** $27+y^{2}-12y=0$ **5)** $6x^{2}+7x-3=0$ **6)** $4y^{2}-18y=-20$

**Solving Quadratic Equations by Taking the Square Root**

If the quadratic equation does not have a linear term, than you do not need to factor to solve the equation. Just isolate the quadratic term and then take the square root of both sides. Remember, when you take a square root you must put a “±” in front of your answer!

**7)** $-30+y^{2}=6$ **8)** $3x^{2}-3=0$ **9)** $40-5y^{2}=-10$

**Writing Quadratic Equations When Given the Roots**

**10)** $3, -5$ **11)** $5(multiplicity 2)$ **12)** $\frac{1}{2}, -3$

©Rasey2017

**Solving Quadratic Equations by Completing the Square (a = 1)**

The process of completing the square allows you to solve a quadratic equation by rewriting it as a perfect square trinomial so that you can take the square root of both sides. This method allows you to solve *all* quadratic equations, not just those that are factorable.

**Steps for solving quadratic equations by completing the square:**

1. Move the constant (c) to the right hand side of the equation.
2. Take half of the coefficient of the linear term, square it, and add it to both sides of the equation.
3. Factor the perfect square trinomial on the left side of the equation and combine the numbers on the right side of the equation.
4. Take the square root of both sides of the equation. (REMEMBER the  on right!)
5. Solve for the variable.

**Examples:** Complete each perfect square trinomial and then factor.

**13)  14) **

**15)  16) **

**Examples:** Solve by completing the square.

**17)  18)  19) **

**You Try:**

**20)  21)  22) **

©Rasey2017

**Notes: Solving Quadratic Equations by Completing the Square\_a≠1**

Previously, we solved quadratic equations by completing the square when the quadratic term had a coefficient of 1. Now, we will solve quadratic equations by completing the square when a ≠ 1. The process is the same with one important additional step… *before* you divide the middle term you must:

**Examples:** Solve each quadratic by completing the square.

**1)  2)  3) **

**You Try:**

**4)  5)  6) **

**Now try this…**

Given the equation, $ax^{2}+bx+c=0$, solve for *x* by completing the square.

©Rasey2017

**\_\_\_\_\_\_\_\_Notes: Solve Using the Quadratic Formula \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

This formula is called the **QUADRATIC FORMULA** and it allows us to solve *any* quadratic equation.

 **The Quadratic Formula**: If $ax^{2}+bx+c=0$, then

**Steps to solve using the Quadratic Formula:**

1. Set the equation equal to zero. (*MUST* be set to zero.)
2. Identify the values of *a*, *b*, and *c* from the equation.
3. Substitute *a*, *b*, and *c* into the quadratic formula.
4. Simplify the expression using the order of operations and rules for simplifying radicals.
5. If the simplified expression has a radical or $i$, then write it as *one* expression with ±.

If there is no radical or $i$, then split into *two* expressions (+ and −) and evaluate each.

**Examples:**

**1)** Solve $x^{2}-10x-24=0$ Solve $x^{2}-10x-24=0$

 By factoring: Using the quadratic formula:

**2)** $y^{2}+1=7y$ **3)** $x^{2}-3x+7=0$

**4)** $4x^{2}=-25+20x$ **5)** $x^{2}-9=0$

©Rasey2017

**\_\_\_\_\_\_\_\_Notes: The DISCRIMINANT\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Notice that the type of answer is determined by the value under the radical, $b^{2}-4ac$. This value is called the **discriminant**. Complete the table to summarize the possible types of solutions.

|  |  |  |
| --- | --- | --- |
| *Value of discriminant* | *Discriminant is a perfect square?* | *Number and Type of Roots* |
| $b^{2}-4ac>0$ (pos.) | Yes |  |
| $b^{2}-4ac>0$ (pos.) | No |  |
| $b^{2}-4ac<0$ (neg.) | -- |  |
| $$b^{2}-4ac=0$$ | -- |  |

**Examples:** Find the values of “k” for which the given equation will have the specified type of roots.

**6)** y2 – 2ky + 7 = 0 ; one double root **7)** x2 - x + k = 0 ; complex roots

**8)** a2 - 4a + k = 0 ; 2 real roots **9)** kx2 + 3x - 1 = 0; one double root

## Summary of How To Solve Quadratic Equations:

## Always look for the GCF first!

1. Divide out common numerical value (not variable)! Or multiply to eliminate fractions!
2. If binomial and both terms have a variable, move to same side and solve by factoring or if one term is a constant, solve by factoring or the square root property.
3. If TRINOMIAL, solve by factoring, completing the square or the Quadratic Formula.

©Rasey2017

**\_\_\_\_\_Notes: Graphing Quadratic Functions \_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

The graph of a quadratic function is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Here are some characteristics of a parabola:

* The highest or lowest point on the parabola is called the \_\_\_\_\_\_\_\_\_\_\_\_\_.
* The \_\_\_\_\_\_\_\_\_\_\_\_ of the vertex point is the maximum or minimum value of the function.
* The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is the VERTICAL line that divides a parabola in half.
* The axis of symmetry (A.O.S.) for any parabola will always be \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The “zeros” are the roots of the equation (the solutions we have been studying!)

 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Vertex Form of a Quadratic:** Vertex:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 $y = a(x – h)^{2} + k$ Axis of Symmetry: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 “a”: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Examples:** Find the vertex and axis of symmetry of each quadratic equation. Also state whether the parabola opens up or down and if it is wider or narrower than $y=x^{2}$

**1)** $y=2\left(x-4\right)^{2}+3$ **2)** $y=\frac{1}{4}\left(x+5\right)^{2}-8$

**3)** $y=\left(x-9\right)^{2}$ **4)** $y=\frac{1}{4}\left(x\right)^{2}-1$

©Rasey2017

**Examples:** Change each equation from standard form to vertex form by completing the square. Then state that vertex and axis of symmetry.

**5)** $y=x^{2}-4x-9$ **6)** $y=2x^{2}-6x-8$

**7)** Write the equation in vertex form of the parabola that has a vertex at (-4, 6) and passes through the point (-1, 9).

**8)** Write the equation in vertex form of the parabola that has a vertex at (2, -3) and passes through the point (4, -1).

**Using the graphing calculator to find the vertex , axis of symmetry, and roots for each quadratic equation below. Then graph the equation and state the domain and range of each.**

**9)** y = x2 + 2x + 1 **10)** y = -6x2 – 12x + 1

Vertex: Vertex:

\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_

A.O.S : A.O.S:

\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_

Roots (Zeros): Roots (Zeros):

\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_

Domain: \_\_\_\_\_\_\_\_\_\_ Range: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Domain: \_\_\_\_\_\_\_\_\_\_\_\_\_\_ Range: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

©Rasey2017

**\_\_\_\_\_Notes: Quadratic Word Problems \_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Example 1:** An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height *s* at time *t* seconds after launch is *s*(*t*) = –4.9*t*2 + 19.6*t* + 58.8, where *s* is in meters.

1. When does the object strike the ground?
2. What was the maximum height the cannon reached?
3. When did the cannon ball reach this maximum height?

 **Example 2:** You have decided that you need a break from Honors Algebra 2 so you and your friend Hermen go to the top of the tallest building around. You toss your algebra book over the edge at the same instant that Herman chucks his book straight down at 48 feet per second. Your book follows the parabolic path modeled by the equation $s\left(t\right)=-16t^{2}+160$ and Herman’s falls at the rate $\left(t\right)=-16t^{2}-48t+160$ . By how many seconds does his book beat yours into the water?

**Example 3:** Suppose you are tossing an apple up to a friend on a third-story balcony. After t seconds, the height of the apple in feet is given by h(t) = -16t2 + 38.4t +.96. Your friend catches the apple just as it reaches its highest point. How long does the apple take to reach your friend, and at what height above the ground does your friend catch it?

**Example 4:** The barber’s profit p each week depends on his charge c per haircut. It is modeled by

p = -200c2 + 2400c – 4700. What price should he charge for the largest profit? What is the max profit?

**Example 5:** The path of a baseball after it has been hit is modeled by the function h(d) = -.0032d2 + d + 3, where h is the height in ft of the baseball and d is the distance in ft the baseball is from home plate. What is the max. height reached by the baseball? How far is the baseball from home plate when it reaches its max. height?

©Rasey2017